Formal Proof:
The Torsional Trefoil Markov Pump Theorem (TTMPT)

Owl Academy
BioChain AI

June 19, 2026

Abstract

The Torsional Trefoil Markov Pump Theorem (TTMPT) establishes a deterministic mathematical framework for a one-way stochastic pump mechanism acting on a longitudinal binding-scalar field. This paper formalizes the 10-step macro-cycle operating over a 12-channel lattice Hilbert space ($\mathcal{H}_{\Lambda_{12}}$). By introducing dynamic transverse variables (π^2 and $\sqrt{\pi}$) as orthogonal heat sinks, formally postulating the projection of Hilbert-space amplitudes into classical Markov probabilities, and utilizing a topological "Tesla-valve" Markovian kernel, we rigorously prove that the system breaks detailed balance. Utilizing explicit stochastic boundary conditions, we demonstrate that the TTMPT generates a strictly positive, continuous net-forward probability current, advancing the binding scalar field while resetting the modular rotational phase to identity.

1 Axiomatic Foundations and State Space

1.1 The Rigorous State Space

The structural bedrock of the system consists of a triple-trefoil skeleton. We rigorously define the localized state space as a 12-channel lattice Hilbert space:

$$\mathcal{H}_{\Lambda_{12}} \cong \mathbb{C}^{12}.$$

The space operates using an explicit basis representing three coupled 4-phase streams (denoted by phase groups α, β, γ). A state vector at macro-step n is defined as the superposition

$$|\psi_n\rangle = \sum_{g\in\{\alpha,\beta,\gamma\}} \sum_{r=1}^{4} c_{g,r}^{(n)} \, |e_{g,r}\rangle.$$

This state vector is subject to the standard normalization constraint ensuring conservation of total probability amplitude:

$$\sum_{g,r} \left| c_{g,r}^{(n)} \right|^2 = 1.$$

1.2 Dynamical Transverse Variables

To govern the continuous spatial scaling and continuous wave energy orthogonal to the central longitudinal binding field B, we introduce a scalar deformation parameter δ alongside continuous topological perturbation modes $\hat{u}(t)$ and $\hat{v}(t)$. The transverse orthogonal foliation variables are rigorously defined as

$$u(t) = \pi^2 + \delta\,\hat{u}(t) \qquad \text{(Outer Manifold Variance)},$$

$$v(t) = \sqrt{\pi} + \delta\,\hat{v}(t) \qquad \text{(Inner Skeleton Winding Parameter)}.$$

These continuously parameterized variables dictate the continuous deformation of the Clifford torus during the stochastic pump cycle. They explicitly define the structural topological resistance function $A_{ij}(t) = A_{ij}\big(u(t), v(t)\big) > 0$ for each graph edge, acting as independent orthogonal heat sinks for reverse-propagating current.

2 Topological Asymmetry and Broken Detailed Balance

2.1 The Postulated Hilbert-to-Markov Probability Bridge

To connect the quantum-like state vectors of $\mathcal{H}_{\Lambda_{12}}$ to classical stochastic dynamics, we explicitly define the projection mapping.

Definition (Classical Markov Projection)**.** The *classical Markov projection* of the Hilbert-space state is the probability vector $p^{(n)}$ with entries

$$p^{(n)}_{g,r} = \big|c^{(n)}_{g,r}\big|^2.$$

Consolidating these probabilities yields a 12-dimensional classical probability vector $p^{(n)} = \big(p^{(n)}_1, \ldots, p^{(n)}_{12}\big)$. We postulate that, under the coarse-grained pump dynamics, this projected probability vector evolves according to a row-stochastic Markov kernel K such that

$$p^{(n+1)} = p^{(n)} K.$$

2.2 Broken Detailed Balance and the Asymmetric Kernel

For a finite Markov chain with stationary distribution π, *detailed balance* holds if $\pi_i K_{ij} = \pi_j K_{ji}$ for all states i, j. To break detailed balance, we define a non-empty oriented forward edge set $E^+ \neq \varnothing$.

Lemma 2.1 (Asymmetric Markovian Kernel)**.** *Suppose that for every $(i,j) \in E^+$ the row-stochastic transition probabilities satisfy*

$$K_{ij} > 0, \qquad K_{ji} = K_{ij}\, e^{-\lambda A_{ij}},$$

where $\lambda > 0$ and $A_{ij} > 0$. If, additionally, the stationary distribution satisfies the structural condition

$$\pi_i > \pi_j\, e^{-\lambda A_{ij}},$$

then it strictly follows that

$$\pi_i K_{ij} - \pi_j K_{ji} > 0.$$

Hence the net probability current $J = \sum_{(i,j)\in E^+} \big(\pi_i K_{ij} - \pi_j K_{ji}\big)$ over the non-empty set is strictly positive.

Proof. The current along any directed edge (i,j) is $J_{ij} = \pi_i K_{ij} - \pi_j K_{ji}$. Substituting the topological suppression rule $K_{ji} = K_{ij}\, e^{-\lambda A_{ij}}$ gives

$$J_{ij} = K_{ij}\big(\pi_i - \pi_j\, e^{-\lambda A_{ij}}\big).$$

Because we have explicitly assumed $K_{ij} > 0$ and $\pi_i > \pi_j\, e^{-\lambda A_{ij}}$, the term in parentheses is strictly positive. Therefore $J_{ij} > 0$ for every edge in E^+. Since $E^+ \neq \varnothing$, there exists at least one edge (i,j) such that $\pi_i K_{ij} \neq \pi_j K_{ji}$, so detailed balance fails. Consequently the net current J over the non-empty set is a sum of strictly positive terms. □

3 Mechanics of the 10-Step Macro-Cycle

3.1 Phase I: Compression and the Frame 5 Solver

Let the 12-channel state lie in a Hilbert space $\mathcal{H} \cong \mathbb{C}^{12}$, and assume $\||\psi_0\rangle\| = 1$. Let $P_1, \ldots, P_4$ be orthogonal projections on $\mathcal{H}$ and let Π be a unitary discrete phase operator. Define the compression operator

$$C = P_4 P_3 \Pi P_2 P_1.$$

Since each P_k is norm non-increasing and Π is norm-preserving, C is norm non-increasing ($\|C|\psi\rangle\| \leq \||\psi\rangle\|$). Assuming $C|\psi_0\rangle \neq 0$, define the normalized pre-pump compressed state by

$$|\psi_{\text{comp}}\rangle = \frac{C|\psi_0\rangle}{\|C|\psi_0\rangle\|} = \frac{P_4 P_3 \Pi P_2 P_1 |\psi_0\rangle}{\|P_4 P_3 \Pi P_2 P_1 |\psi_0\rangle\|}.$$

Then $\||\psi_{\text{comp}}\rangle\| = 1$. The Frame 5 Algebraic Solver provides the geometric phase-lock parameter

$$s = 1 - \frac{(Q_\alpha + Q_\beta - Q_\gamma)^2}{4\, Q_\alpha Q_\beta}, \qquad Q_\alpha, Q_\beta > 0.$$

Setting $s = \sin^2\left(\frac{2\pi}{3}\right) = \frac{3}{4}$ imposes the symmetric 120° phase-lock condition. This geometric condition motivates the topology of the cycle, while the rigorous contraction mechanism is supplied by the Markov kernel assumptions in the pump phase.

3.2 Phase II: Core Pump Stroke and Time Parameterization

During the pump interval $t \in [t_5, t_7]$, let $K(t)$ be a continuously parameterized family of stochastic transition matrices. For example, in the row-stochastic convention, $K_{ij}(t) \geq 0$ and $\sum_j K_{ij}(t) = 1$. The compressed state induces a probability vector

$$p_i^{\text{comp}} = \left|\langle e_i \,|\, \psi_{\text{comp}}\rangle\right|^2.$$

The Markov kernel evolves this probability layer during the pump stroke. If the larger theorem requires strict contraction, assume there exists $0 \leq \lambda < 1$ such that $\|pK(t) - qK(t)\|_1 \leq \lambda\, \|p - q\|_1$ for all admissible probability vectors p, q and all $t \in [t_5, t_7]$. Let S_1, S_2, S_3 be unitary rotational operators ($S_k^* S_k = I$, so $S_k^{-1} = S_k^*$). Assuming $\sum_{k=1}^{3}(S_k + S_k^{-1})|\psi_0\rangle \neq 0$, define the normalized quaternion centroid by

$$|\Psi_4\rangle = \frac{\sum_{k=1}^{3}(S_k + S_k^{-1})|\psi_0\rangle}{\|\sum_{k=1}^{3}(S_k + S_k^{-1})|\psi_0\rangle\|}.$$

The active current $J(t)$ is then interpreted as a parameterized flow directed toward $|\Psi_4\rangle$.

3.3 Phase III: Double Toroidal Collapse

Let $\big(u(t), v(t)\big) \in \mathbb{T}^2$ parameterize the toroidal collapse, and let $A_{ij}\colon \mathbb{T}^2 \to \mathbb{R}_{\geq 0}$ be continuous topological resistance functions. Introduce a reverse-energy functional $E_{\text{rev}}(t) \geq 0$. The collapse phase is dissipative if

$$\frac{d}{dt}\, E_{\text{rev}}(t) = -\sum_{i,j} A_{ij}\big(u(t), v(t)\big)\, R_{ij}(t)^2 \leq 0.$$

It is strictly dissipative whenever at least one active resistance mode satisfies $A_{ij}\big(u(t), v(t)\big)\, R_{ij}(t)^2 > 0$. Thus reverse variance is dissipated into transverse modes while the longitudinal field remains protected by a separately imposed conservation or lower-bound condition.

3.4 Phase IV: Encapsulation and Phase Reset

Let $\Theta_k \in \mathbb{R}/2\pi\mathbb{Z}$ and define the modular phase update by $\Theta_{k+1} = \Theta_k + \omega_k$. After one full 10-step macro-cycle,

$$\Theta_{10} = \Theta_0 + \sum_{k=0}^{9} \omega_k.$$

Encapsulation requires the residual modular rotation to vanish: $\sum_{k=0}^{9} \omega_k \equiv 0 \pmod{2\pi}$. Equivalently, $\Theta_{10} \equiv \Theta_0 \pmod{2\pi}$.

4 The Torsional Trefoil Markov Pump Theorem (TTMPT)

With the axiomatic mechanics established, we formalize the principal theorem.

Theorem 4.1 (TTMPT). *Let Λ_{12} be a finite state space with 12 states, and let $E^+ \subseteq \Lambda_{12} \times \Lambda_{12}$ be a non-empty oriented forward edge set ($E^+ \neq \varnothing$). Let $[t_5, t_7]$ be a non-degenerate compact interval with $t_7 > t_5$. For each $t \in [t_5, t_7]$, let $K(t)$ be a continuously parameterized family of discrete-time row-stochastic, irreducible, aperiodic transition matrices. Assume that $K_{ij}(t)$, the unique stationary distribution $\pi_i(t)$, and the topological resistance function $A_{ij}(t) = A_{ij}\big(u(t), v(t)\big)$ are continuous in t, satisfying*

$$\pi(t)K(t) = \pi(t), \qquad \sum_i \pi_i(t) = 1, \qquad \pi_i(t) > 0.$$

Assume that for every $(i, j) \in E^+$ and every $t \in [t_5, t_7]$,

$$K_{ij}(t) > 0, \qquad K_{ji}(t) = K_{ij}(t)\, e^{-\lambda A_{ij}(t)},$$

where $\lambda > 0$ and $A_{ij}(t) > 0$, and assume

$$\pi_i(t) > \pi_j(t)\, e^{-\lambda A_{ij}(t)}.$$

Define the continuous forward probability current by

$$J(t) = \sum_{(i,j)\in E^+} \big(\pi_i(t)K_{ij}(t) - \pi_j(t)K_{ji}(t)\big).$$

Then $J(t) > 0$ for all $t \in [t_5, t_7]$. Consequently, if B satisfies $\dfrac{dB}{dt} = J(t)$, then

$$B(t_7) > B(t_5).$$

Furthermore, if the discrete phase update is $\Theta_{k+1} = \Theta_k + \omega_k$ and satisfies the closure condition $\sum_{k=0}^{9} \omega_k \equiv 0 \pmod{2\pi}$, then

$$\Theta_{10} \equiv \Theta_0 \pmod{2\pi}.$$

Proof. For each $(i, j) \in E^+$ at any time $t \in [t_5, t_7]$, using the topological suppression rule,

$$\begin{aligned}\pi_i(t)K_{ij}(t) - \pi_j(t)K_{ji}(t) &= \pi_i(t)K_{ij}(t) - \pi_j(t)K_{ij}(t)\, e^{-\lambda A_{ij}(t)} \\ &= K_{ij}(t)\big(\pi_i(t) - \pi_j(t)\, e^{-\lambda A_{ij}(t)}\big).\end{aligned}$$

Since $\pi_i(t) > \pi_j(t)\, e^{-\lambda A_{ij}(t)}$ by hypothesis, the quantity in parentheses is strictly positive, and factoring out the strictly positive $K_{ij}(t)$ yields

$$\pi_i(t)K_{ij}(t) - \pi_j(t)K_{ji}(t) > 0.$$

Because E^+ is non-empty ($E^+ \neq \varnothing$), there exists at least one edge (i, j) such that $\pi_i(t)K_{ij}(t) \neq \pi_j(t)K_{ji}(t)$, so detailed balance fails at each $t \in [t_5, t_7]$. Summing over the forward edge set gives a strictly positive continuous net current:

$$J(t) = \sum_{(i,j)\in E^+} \big(\pi_i(t)K_{ij}(t) - \pi_j(t)K_{ji}(t)\big) > 0.$$

Given that the scalar field evolves as $\frac{dB}{dt} = J(t)$, we integrate over the non-degenerate interval $[t_5, t_7]$:

$$B(t_7) - B(t_5) = \int_{t_5}^{t_7} J(t)\, dt.$$

Because $J(t) > 0$ and is continuous throughout the entire interval, and $t_7 > t_5$, the definite integral is strictly positive:

$$\int_{t_5}^{t_7} J(t)\, dt > 0.$$

Hence $B(t_7) - B(t_5) > 0$, demonstrating $B(t_7) > B(t_5)$.

Finally, iterating the discrete phase updates through the 10-step macro-cycle gives

$$\Theta_{10} = \Theta_0 + \sum_{k=0}^{9} \omega_k.$$

Using the phase-closure constraint $\sum_{k=0}^{9} \omega_k \equiv 0 \pmod{2\pi}$, we obtain

$$\Theta_{10} \equiv \Theta_0 \pmod{2\pi}.$$

Therefore, one full macro-cycle produces strictly positive advancement of the scalar field B while successfully resetting the modular phase to identity. □

End of Proof.